OPTIMAL ORDER QUANTITY ASSUMING THE COMPONENT PART QUANTITY IS A RANDOM VARIABLE.

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Abstract

This paper shows how to estimate the optimal order quantity for unique batch assemblies given that the component part quantity is a random variable. The population consists of customer specified, dated assemblies with unique composition and application. The probability model assumes that the quantity of the component parts is a random variable. This model uses four unknown parameters that are needed to estimate the optimal order quantity. The results demonstrate methods of parametric analysis for evaluation of management intervention effectiveness.

Introduction

In the Printing Industry, unique component parts are assembled to produce books, magazines and catalogs for a specific distribution date. After the distribution date, the remaining assemblies and component parts have no value to the customer. In the assembly process, the total product is partitioned into a finite number of unique component parts. All component parts are needed to complete the final product. The component parts are unique to each specific finished good. Therefore, when any component part is completely consumed, no additional total products can be produced.

Due to manufacturing methods, the quantity measurement of these component parts is not precisely known and is represented as a random variable. In addition, spoilage and loss of component parts during the remaining manufacturing processes cause fewer component parts to be combined into usable final assemblies. Finally, due to limited production time and the relatively high cost of set-up, a second production run is avoided if at all possible. The determination of the optimal order quantity is based upon several model parameters including set-up cost, component part cost, customer demand range, number of component parts per finished good, selling price and the variance and shape of the distribution of the component part quantity.

Previous Research

Yao [4] analyzed a system to determine order quantities for an assembly system with random yields. A random variable R_i was used to represent the yield ratio of the delivered quantity divided by the ordered quantity. The setup cost and component parts cost were combined to provide a cost function of the run quantity. The net finished good quantity was the minimum of the net yield of each of the component parts.

Kuriyama [2] discussed a two-echelon purchasing / production system. In this analysis, a known quantity of component parts was purchased. Due to deterioration and failure both in stock and production stages the quantity was subject to losses. The goal of the analysis was to determine EOQ (economic order quantity) and EPQ (economic production quantity). The costs of purchase, setup, holding, decay and failure were considered.

Thomopoulos [3] modeled optimum order quantity for dated items. In this model, the order quantity was a constant, and the demand was expressed as a probability distribution. Quantitative methods were used to create reference tables. These tables were arranged according to parameter values with the normalized, optimal order quantity and profit as resultant answers.

Model Development

Throughout the production process, the conforming quantities of the individual component parts are not exactly known. For purposes of this analysis, it is assumed that the actual quantity produced of part i, Q_i, is distributed with a mean of μ and a standard deviation σ. In this study, the mean and standard deviation are the same for each component part. The mean μ is equal to the press order quantity of component parts that the printer has specified rather than the quantity of final assemblies ordered by the customer.

The actual quantity of assemblies produced is equal to the minimum of the conforming quantities of each of the component parts. If the quantity of assemblies produced is greater than the maximum customer order, only the maximum customer order quantity is sold. If the quantity of assemblies produced is less than or equal to the maximum customer order quantity, all assemblies produced so far are sold. When the quantity of assemblies produced is less than the specified minimum customer order quantity, a second production run is necessary. This second production run requires another component part setup and an assembly setup as well as the related production cycle.

The input variables are a function of the customer and the manufacturing process. The customer inputs are: maximum and minimum customer demand for finished goods (D_{max} and D_{min}), the number of component parts per finished good (n) and the finished good selling price (S). The production process inputs are: set up cost (C_s), component part cost (C_p), quantity of finished goods produced (N) and quantity of finished goods sold (N_s).

The normalizing ratios are component part cost ratio (R_1), setup cost ratio (R_2), customer demand ratio (R_3), order quantity ratio (q), profit ratio (r) and variation ratio (rov). The computation of these ratios are determined by the following equations:

$R_1 = n\,C_p / S$ (1)

$R_2 = C_s / (S\,D_{max})$ (2)

$R_3 = D_{min} / D_{max}$ (3)

$q = Q / D_{max}$ (4)

$r = E(\Pi) / (S\,D_{max})$ (5)

$rov = \sigma / D_{max}$ (6)

A main purpose of this research is to analyze different methods used to evaluate the printer's internal optimal order quantity of component parts. Three assumptions are used in building the model:

(1) The finished good requires a specific number of unique parts of equal cost (The component parts are uniquely different but manufacturing costs are assumed to be identical.);

(2) The costs of the initial setup for the assemblies and component parts are known;

(3) The printer's selling price is constant within the minimum and maximum customer order quantity range.

Calculation of the Cumulative Distribution Function

The actual number of assemblies N is a discrete distribution with positive integer values. A goal is to find P(N), which is the probability distribution of N. To do this, the probability for each value of N is calculated. The probability that N is equal to some value x, P(N = x), is equal to the sum of the probabilities that the actual quantity of one part is equal to x, $P(Q_i = x)$; for some i = 1, 2, ..., n multiplied by the probability that the actual quantity for all of the other parts is equal to or greater than x, $P(Q_{j \neq i} \geq x)$, for all j = 1, 2,..., n. Since all quantities Q_i are independent, the individual probabilities are multiplied together to find the joint probability.

$$P(N = x) = \sum_{j=1}^{n} \left(P(Q_j = x) \prod_{i \neq j}^{n} P(Q_i \geq x)\right) \quad (7)$$

Calculation of Expected Profit

The expected profit $E(\Pi)$ is equal to the expected revenue E(R) minus the expected total costs E(TC), i.e.,

$$E(\Pi) = E(R) - E(TC) \quad (8)$$

The expected revenue is equal to the price multiplied by the expected number of finished goods sold.

$$E(R) = S\, E(N_s) \quad (9)$$

Using the probability for each state and the number of assemblies sold (N_s), the expected number of assemblies sold is obtained as below:

$$E(N_s) = P(N \geq D_{max})\, D_{max} + \sum_{N=D_{min}}^{D_{max}-1} N\, P(N) + P(N < D_{min})\, D_{min} \quad (10)$$

Substitution of terms yields the following normalized equation:

$$R = P(N \geq D_{max}) + (1 / D_{max}) \sum_{N=D_{min}}^{D_{max}-1} N\, P(N) + R_3\, P(N < D_{min}) - R_2\, [1 + P(N < D_{min})] -$$

$$R_1\, [q - (1 / D_{max}) \sum_{i=0}^{D_{min}} P(Q_i)\, (D_{min} - Q_i)] \quad (11)$$

Optimal Order Quantity Calculations

Based upon Equation 11, Allen [1] created numerical approximation models for optimal r and q. To demonstrate the use of these models, an example problem is analyzed. Suppose a customer order quantity ranges from 4,750 to 5,000 finished goods. Assume also each finished good requires 12 component parts of equal cost of $0.10 each. The combined assembly setup costs is $1,500.00 and the selling price per finished good is $3.00. The standard deviation of the component part quantity is 250. The probability distribution for the component part quantity is approximately normal. The parameters n, R_1, R_2, R_3, and rov are needed to find the optimal component part order quantity Q and the expected profit $E(\Pi)$. These are obtained as below:

$n = 12$

$R_1 = n C_p / S = 12 \times \$ 0.10 / \$ 3.00 = 0.40$

$R_2 = C_s / (S D_{max}) = \$ 1,500.00 / \$ 3.00 \times 5,000 = 0.10$

$R_3 = D_{min} / D_{max} = 4750 / 5000 = 0.95$

$rov = \sigma / D_{max} = 250 / 5000 = 0.050$

The normalized results are taken from the numerical models. These become q = 1.129 and r = .440. Converting to the optimal component part order quantity:

$Q = q D_{max} = 1.129 \times 5,000$

Q = 5,645 parts

and the expected profit:

$E(\Pi) = r S D_{max} = .440 \times \$ 3.00 \times 5,000$

$E(\Pi) = \$ 6,600$

Parametric Analysis

The primary objective of this study, as illustrated in the previous example, is to determine the order quantity that will optimize profits for a given finished good. For the purposes of process optimization, several other questions are also significant. For example, an investment in process technology could reduce the component part quantity count variation, thereby reducing the value of the parameter rov. The question that can be answered is: "For a given set of process parameter conditions, what is the economic value of a reduction of the parameter rov?" Based upon this analysis, a project internal rate of return can be calculated to determine economic feasibility of this investment to reduce the rov.

Using the previous example, the results will improve as a result of the reduction in process variation. Assuming the parameter rov is reduced to 0.005, the profit factor r increases from 0.4400 to 0.4939. Thus the expected profit increases from $6,600 to $7,408 per job.

Conclusions

The final variables are normalized and transformed to dimensionless ratios for universal application. This allows tables to be generalized for variations in size and scope. Although the base problem is related to the printing industry, the variable characteristics used in this study may be applied to any batch assembly process with an uncertain component parts quantity. The traditional coefficient of variation is modified to the ratio of variation to facilitate the comparison to the maximum customer order quantity.

The quantitative techniques are developed on Microsoft Excel so that variations can be developed on a standard personal computer with typical office software. The current processing capability, speed and memory of computer systems allow a wide range of complex quantitative solutions with minimal total calculation time. The wide range of Excel statistical functions allows numerous variations of the base problem. This feature permits further development of these concepts by individuals who have access to a personal computer with Microsoft Excel software.

References

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