Multi-Item Ordering Decision with Target Discount Contract

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ABSTRACT

This paper concerns a multi-item discount problem by presenting a heuristic that determines the replenishment of all items; whether to replenish items individually or jointly given an all-unit discount contract from a single supplier. When buying individually, the economic order quantity is used; as a group, the minimum time supply that satisfies the total buy constraint is used to find the buy quantity. The decision is made at a point in time by choosing the alternative that provides the lower comparative cost. With the simplicity and performance, this heuristic can efficiently solve large size problems.

INTRODUCTION

A multi-item discount problem deals with a buyer's replenishment decision when a supplier offers a reduction in unit cost for orders that reach a supplier's discount conditions such as minimum order quantity, minimum order cost, or minimum order weight. The savings can be achieved by coordinating replenishment of groups of items. The coordinated replenishment decision is influenced by the lower unit cost, lower transportation cost, lower ordering cost, higher inventory cost, and lower flexibility. In this paper, several questions will be answered: 1) when to order, 2) how much to order, 3) whether to order individually or jointly, 4) which item to be included in a group, and 5) how to allocate the order quantity among the items in a group. This paper is part of the thesis of Ornlatcha Sivarak for the Ph.D. in Management Science at the Stuart School of Business at the Illinois Institute of Technology.

A number of studies have been done regarding the multiple items quantity discount problem. Pirkul and Aras (1985) obtained optimal solutions of small size capacitated multiple item ordering problems with all-unit quantity discount based on a Lagrangian relaxation approach. Because of exhaustive computation of the large size problems, a heuristic algorithm is proposed. Benton (1991) developed a heuristic to evaluate alternative discount schedule of a multiple item quantity discount problem with multiple suppliers and resource limitations. The heuristic procedure using Lagrangian relaxation is also proposed to solve problem of multiple supplier with discount schedule. Rubin and Benton (1993) extended the work of Benton (1991) and presented an optimal solution approach by using non-linear programming and partial enumeration for an all-units discounts problem. Güder, Zydiak, and Chaudhry (1995) generated non-stationary policies that always provide better solutions to those generated by the Lagrangian approach and sometimes provide better to those generated by the fixed cycle method. The basic idea of this non-stationary approach is to order as much as possible without exceeding the EOQ value and the available spaces. Güder et al (1996) presented an extended heuristic algorithm of Pirkul and Aras (1985) to solve the large instances of the problem. Güder and Zydiak (1997) developed a non-stationary method based on Güder et al (1995). This approach relaxes the restriction in stationary methods that order quantities must remain constant over time. The result is preferred to stationary approaches. Güder and Zydiak (2000) demonstrated a fixed cycle approach, where the time between order replenishments for all items is the same, to solve multi-item with the quantity discount problem. You (2005) addressed a heuristic approach for multiple item and location ordering problem with quantity discount and capacity constraint through a geographically distributed vending machine problem. Due to the computational complexity in deriving the exact optimal solutions, the heuristic that find the solution that is as effective as the optimal solution is proposed.

In this study, we present a heuristic that compares the costs between two alternatives, the individual buy and the joint buy. The individual order quantity for each item is assumed to be the typical economic order quantity (EOQ) that theoretically provides the minimum cost. The discount given by the supplier is an all-units discount. The order quantities of all items are combined in order to reach the discount condition. The excess quantities from regular EOQ, if any, are distributed among the items based on their depletion rates. The cost of both alternatives is calculated based on the equal amount of order quantity. It is clear that the excess quantity from the EOQ will result in more expensive holding cost. If the total savings from the supplier compensates this excess holding cost, then we will choose to order at the supplier's minimum requirement.

NOTATIONS

Given Parameters:

i	a number of items, $i = 1, 2, \ldots, n$
j	a discount level, $j = 1, 2, \ldots, m$
f_{1i}	an average one-month demand forecast for item i
A_i	an annual demand forecast for item i
L_i	a lead time in months for item i
C_{oi}	a cost per order for item i
C_i	a unit cost for item i
C_{si}	a fixed excess storage cost for item i
h	an annual holding rate
e_i	a variable excess storage cost per unit for item i
$ohoo_i$	an on-hand plus on-order inventory for item i
ss_i	a safety stock for item i
F_{Li}	a lead time demand forecast for item i
op_i	an order point for item i
EOQ_i	an economic order quantity for item i
ol_i	an order level for item i
DQ_j	a target dollar discount j
UQ_j	a target quantity discount j
d_j	a target discount rate j

Decision Variables:

el_{ji}	an excess storage level in discount level j for item i
q_i	a cycle buy quantity in economic cycle for item i
q'_i	a current buy quantity in economic cycle for item i
Q_{ji}	a cycle buy quantity in target cycle in discount level j for item i
Q'_{ji}	a current buy quantity in target cycle in discount level j for item i
Qe_{ji}	an excess quantity in target cycle in discount level j for item i
τ_i	an individual time supply for item i
τ_{Qj}	a target discount time supply in discount level j
τ_{Qji}	a target discount time supply in discount level j for item i

S_{Qj}	a target order-plus-stock above safety stock in discount level j
S_i	an individual order-plus-stock above safety stock for item i
$F_{\tau Qji}$	a target discount time supply demand forecast in discount level j for item i
δ_{ji}	a (0, 1) identifier in target discount time supply calculation.
	$\delta_{ji} = 0$ if $\tau_i > \tau_{Qj}$ and $ohoo_i > op_i$; $\delta_{ji} = 1$ if $\tau_i \le \tau_{Qj}$
β_i	a (0, 1) identifier in on-hand plus on-order units above the order point calculation.
	$\beta_i = 0$ if $ohoo_i \le op_i$; $\beta_i = 1$ if $ohoo_i > op_i$
α_{ji}	a (0, 1) identifier in excess quantity calculation Qe_{ji}. $\alpha_{ji} = 0$ if $Qe_{ji} = 0$; $\alpha_{ji} = 1$ if $Qe_{ji} > 0$
$K(q'_i)$	an economic cycle cost for item i
$\hat{K}(Q'_{ji})$	an adjusted economic cycle cost in discount level j for item i
$K(Q'_{ji})$	a target discount cycle cost in discount level j for item i

HEURISTIC

The proposed heuristic applies to inventory system with item $i = 1, 2, ..., n$ which normally replenished from a single supplier. The demand of each item is assumed to be horizontal. Because of the unknown demand in the future, the demand for each item is given from the estimate demand forecast. The procurement lead time is fixed and the replenishment arrives in one lump sum. No shortage or backorders are allowed. The target discount contract given by a supplier is a price schedule that contains target discount either in units UQ_j, dollars DQ_j, or weight WQ_j and its corresponding target discount rate d_j. The discount rate is $d_j = 1 - \text{percent discount}$. The price schedule breaks into each specified target discount level $j = 1, 2, ..., m$. The target discount discussed is the all-unit target discount where the discount is applied to all units if the order is qualified for the discount.

This heuristic adopts the individual buy policy and compares it with the following joint buy policy to support an order decision. It divides the calculation into two cycles, economic cycle and target discount cycle. In economic cycle, each item assumes an individual buy policy where each item has its own EOQ, order point, order level, and review time. The review time for each item can be periodic or continuous according to buyer's individual buy policy. In target discount cycle, the discount offered by a supplier is allocated among items in a target discount time supply group by balancing the individual time supply of all items. The number of items included in a group is determined by its individual time supply τ_i and the target discount time supply of a group τ_{Qj} as well as its on-hand plus on-order level $ohoo_i$. The ordering decision is made at a point in time as a snapshot. This can be every week, every month, or etc. The decision determines whether to order one item at a time as needed in an individual buy or to order all items together in a joint buy.

In both economic cycle and target cycle, buy quantities and buy costs are determined. In each cycle, there are two buy quantities, a cycle buy and a current buy. The cycle buy quantities are the quantities to buy when the on-hand plus on-order is at the order point. The current buy quantities are determined by the actual on-hand plus on-order level at the review time.

The heuristic is explained in the following three steps:

Step 1: Economic cycle calculation

In economic cycle, cycle buy quantity q_i, current buy quantity q'_i, individual time supply τ_i and total cost of each item $K(q'_i)$ are determined as if each item is ordered separately. The individual time supply τ_i is a length of time in months where a current buy quantity in economic cycle and an on-hand plus on-order beyond the safety stock of an item reach the safety stock. The economic cycle cost of each item $K(q'_i)$ is expressed as order cost, holding cost and material cost. The following calculations are required.

1.1 For each item $i = 1, 2, ..., n$, do step 1.2 – 1.4

1.2 Find buy quantities in economic cycle

1.2.1 Economic order quantity is $EOQ_i = \sqrt{\dfrac{2C_{oi}A_i}{C_i h_i}}$

1.2.2 Safety stock is $ss_1 = w \times f_{1i}$

Note: A safety stock factor w is assumed to be 0.5. It is beyond the scope of this paper to describe what method is used to calculate safety stock. See Thomopoulos, N.T. (1990) on how to determine the safety stock.

1.2.3 Lead time demand forecast is $F_{Li} = L_i \times f_{1i}$

1.2.4 Order point is $op_i = ss_i + F_{Li}$

1.2.5 Order level is $ol_i = op_i + EOQ_i$

1.2.6 Cycle buy quantity in economic cycle is $q_i = ol_i - op_i$

1.2.7 Current buy quantity in economic cycle is $q'_i \;\; = \begin{bmatrix} 0 & \text{if } ohoo_i > op_i \\ ol_i - ohoo_i & \text{if } ohoo_i \le op_i \end{bmatrix}$

1.3 Find the individual time supply τ_i

$$S_i = q'_i + ohoo_i - ss_i$$

$$\tau_i = \frac{S_i}{f_{1i}}$$

Note: $q'_i > 0$ only if the current $ohoo_i \le op_i$

1.4 Economic cycle cost of item i is

$$K(q'_i) = \begin{bmatrix} C_{oi} + C_i h \dfrac{q_i}{A_i}\dfrac{q_i}{2} + C_i q'_i & \text{if } q'_i > 0 \\ 0 & \text{if } q'_i = 0 \end{bmatrix}$$

Total economic cycle cost of all items is $\sum_{i=1}^{n} K(q'_i)$

Step 2: Target discount cycle calculation

In target discount cycle, besides the cycle buy quantity Q_{ji} and the current buy quantity Q'_{ji}, the excess quantity Qe_{ji} is also required. This excess quantity Qe_{ji} may incur in target cycle in order to satisfy the discount offered under the target discount contract. It is the excess quantity beyond the order level. We assume that order quantity above the order level is required extra storage. The target discount time supply τ_{Qj} is used for the calculation of the buy quantities in target cycle. The target discount time supply τ_{Qj} measures a length of time in months where a target discount and an on-hand plus on-order beyond the safety stock of items in a target discount time supply group reach their safety stocks. Items, with the individual time supply higher than the target discount time supply $\tau_i > \tau_{Qj}$ and on-hand plus on-order higher than the order point $ohoo_i > op_i$, are excluded from the target discount time supply group during the readjustment of target discount time supply. This is because those items have high excess on-hand plus on-order that the model decides not to buy more of that item for now. The target discount cycle cost of each item $K(Q'_{ji})$ is expressed as order cost, holding cost, buy cost, and excess cost. The excess storage cost is added here since we assume that the warehouse capacity is only enough for order quantities provided by EOQ.

2.1 Find buy quantities in target cycle

2.1.1 Find a target discount time supply τ_{Qj} for each discount level $j = 1, 2, ..., m$

$$S_{Qj} = DQ_j + \sum_{i=1}^{n} C_i ohoo_i - \sum_{i=1}^{n} C_i ss_i$$

$$\tau_{Qj} = \frac{S_{Qj}}{\sum_{i=1}^{n} C_i f_{1i}}$$

Note: when the supplier gives discount to minimum quantities instead of minimum cost, the target dollar discount DQ_j is replaced by target quantity discount UQ_j and the unit cost C_i of each item in this equation is set to 1.

2.1.2 For each discount level $j = 1, 2, ..., m$ and each item $i = 1, 2, ..., n$, do step 2.1.3 to 2.1.8

2.1.3 Compare the individual time supply of each item τ_i, acquired in step 1, to the target discount time supply in each target discount level τ_{Qj} and readjust the target discount time supply until no item with higher individual time supply than the target discount time supply $\tau_i > \tau_{Qj}$ and an on-hand plus on-order higher than an order point $ohoo_i > op_i$ is included in a group.

2.1.3.1 Set identifier used in the recalculation of target discount time supply

$$\delta_{ji} = \begin{bmatrix} 0 & \text{if } \tau_i > \tau_{Qj} \text{ and } ohoo_i > op_i \\ 1 & \text{if } \tau_i > \tau_{Qj} \text{ and } ohoo_i \le op_i \text{ or } \tau_i \le \tau_{Qj} \end{bmatrix}$$

2.1.3.2 Recalculate target discount time supply τ_{Qj}

$$S_{Qj} = DQ_j + \sum_{i=1}^{n} \delta_{ji} C_i ohoo_i - \sum_{i=1}^{n} \delta_{ji} C_i ss_i$$

$$\tau_{Qj} = \frac{S_{Qj}}{\sum_{i=1}^{n} \delta_{ji} C_i f_{1i}}$$

2.1.4 Target discount time supply in discount level j for item i is

$$\tau_{Qji} = \begin{bmatrix} 0 & \text{if } \delta_{ji} = 0 \\ \tau_{Qj} & \text{if } \delta_{ji} = 1 \end{bmatrix}$$

2.1.5 Target discount time supply demand forecast in discount level j for item i is $F_{\tau Qji} = \tau_{Qji} \times f_{1i}$

2.1.6 Current buy quantity in target cycle is

$$Q'_{ji} = \begin{bmatrix} 0 & \text{if } \delta_{ji} = 0 \\ F_{\tau Qji} - ohoo_i + ss_i & \text{if } \delta_{ji} = 1 \end{bmatrix}$$

2.1.7 Cycle buy quantity in target cycle is

$$Q_{ji} = \begin{bmatrix} Q'_{ji} & \text{if } q'_i = 0 \\ Q'_{ji} - (q'_i - q_i) & \text{if } q'_i > 0 \end{bmatrix}$$

2.1.8 Find the excess quantity in target cycle

2.1.8.1 Excess level is $el_{ji} = ohoo_i + Q'_{ji}$

2.1.8.2 Excess quantity is

$$Qe_{ji} = \begin{bmatrix} el_{ji} - ol_i & \text{if} \quad ohoo_i \le ol_i < el_{ji} \\ el_{ji} - ohoo_i & \text{if} \quad ol_i < ohoo_i < el_{ji} \\ 0 & \text{if} \quad el_{ji} \le ol_i \end{bmatrix}$$

2.2 For each discount level $j = 1, 2, \ldots\ldots, m$ and each item $i = 1, 2, \ldots\ldots, n$, target cycle cost of item i is

$$K(Q'_{ji}) = \begin{bmatrix} C_{oi} + \left[\left(d_j C_i h \frac{Q_{ji}}{A_i}\frac{Q_{ji}}{2}\right) + \beta_i\left(d_j C_i h \frac{(ohoo_i - op_i)}{A_i} Q_{ji}\right)\right] + d_j C_i Q'_{ji} + \alpha_{ji}\left[C_{si} + d_j C_i e_i \frac{Qe_{ji}}{A}\frac{Qe_{ji}}{2}\right] & \text{if } Q'_{ji} > 0 \\ 0 & \text{if } Q'_{ji} = 0 \end{bmatrix}$$

Total target cycle cost of all items in a target discount time supply group is $\sum_{i=1}^{n} K(Q'_{ji})$

Note:

β_i a (0, 1) identifier used in the calculation of on-hand plus on-order units above the order point.
$\beta_i = 0 \quad \text{if } ohoo_i \le op_i$; $\beta_i = 1 \quad \text{if } ohoo_i > op_i$

α_{ji} a (0, 1) identifier used in the calculation of target excess quantity Qe_{ji}. $\alpha_{ji} = 0 \quad \text{if } Qe_{ji} = 0$;
$\alpha_{ji} = 1 \quad \text{if } Qe_{ji} > 0$

Step 3: Comparison between economic cycle and target discount cycle

In step 3, before comparing the total cost of two cycles, the economic cycle cost is adjusted because the buy quantities in economic cycle may not equal to the buy quantities in target cycle. The order cost and the holding cost in economic cycle are multiplied by $\frac{Q_{ji}}{q_i}$ while the material cost is computed using the current buy quantity in target cycle Q'_{ji}. These are done so that the costs of equal quantities are compared. Then, the adjusted economic cycle cost is compared to the target cycle cost to find the cycle that provides lower cost.

3.1 For each discount level $j = 1, 2, \ldots\ldots, m$ and each item $i = 1, 2, \ldots\ldots, n$, the adjusted economic cycle cost of item i is

$$\hat{K}(Q'_{ji}) = \begin{bmatrix} C_{oi}\frac{Q_{ji}}{q_i} + C_i h \frac{q_i}{A_i}\frac{q_i}{2}\frac{Q_{ji}}{q_i} + C_i Q'_{ji} & \text{if } Q'_{ji} > 0 \\ 0 & \text{if } Q'_{ji} = 0 \end{bmatrix}$$

The total adjusted economic cycle cost of all items in a target discount time supply group is $\sum_{i=1}^{n} \hat{K}(Q'_{ji})$

3.2 The buy decision is made according to the lowest cost of all discount levels and the current buy quantities of the cycle that provides the lower cost is placed.

3.2.1 The lowest cost of all discount levels is $\underset{j=1,2,\ldots,n}{MIN}\left\{\sum_{i=1}^{n}\hat{K}(Q'_{ji}), \sum_{i=1}^{n}\hat{K}(Q'_{ji})\right\}$

3.2.2 If the adjusted economic cycle cost is less than or equal to the target cycle cost, the current buy quantities in economic cycle of all items q'_i will be placed. This means that the individual buy for each item is chosen. If the adjusted economic cycle cost is higher than the target cycle cost, the current buy quantity in target cycle of all items Q'_{ji} will be placed. This means that all items are bought jointly.

COMPUTATIONAL RESULTS

To demonstrate the proposed heuristic results, the total costs of all items are compared using the ratio of the total target discount cycle cost $\sum_{i=1}^{n} K(Q'_{ji})$ to total adjusted economic cycle cost $\sum_{i=1}^{n} \hat{K}(Q'_{ji})$. We generate average one-month demand forecasts f_{1i} for 1,000 items using uniform random number in a range of 5-105 units. The order cost C_{oi}, annual holding rate h, fixed storage cost C_{si}, and variable storage rate e_i are the same for all items $i = 1, 2, ..., 1{,}000$. The order cost C_{oi} is \$10 for each item ordered and the fixed storage cost C_{si} is \$100 for each item that need excess storage space. The annual holding rate h is 30% and the variable storage rate e_i is 20 % of the unit cost. The unit costs C_i are generated in a range of \$1-\$50 using uniform random number. We also assume that the on-hand plus on-order $ohoo_i$ of all items are at the order point op_i and the lead time in months L_i is 50%. Table 1 shows the change in ratios to the change in target quantity discount levels UQ_j and rates d_i. The ratio below 1.0 shows that the target discount cycle provides a lower total cost than that of economic cycle cost.

Table 1: The ratio of the total target discount cycle cost to total adjusted economic cycle cost $\dfrac{\sum_{i=1}^{n} K(Q'_{ji})}{\sum_{i=1}^{n} \hat{K}(Q'_{ji})}$

Target quantity discount (UQ_j)	Target discount rate (d_j)											
	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7750	0.7500	0.7250	0.7000
486,403	1.1374	1.1084	1.0795	1.0505	1.0216	0.9927	0.9637	0.9348	0.9058	0.8769	0.8479	0.8190
324,269	1.0816	1.0542	1.0268	0.9994	0.9720	0.9446	0.9172	0.8898	0.8624	0.8350	0.8076	0.7802
243,202	1.0560	1.0293	1.0027	0.9761	0.9494	0.9228	0.8962	0.8695	0.8429	0.8162	0.7896	0.7630
194,561	1.0423	1.0161	0.9899	0.9638	0.9376	0.9114	0.8852	0.8590	0.8328	0.8067	0.7805	0.7543
162,134	1.0343	1.0084	0.9825	0.9566	0.9308	0.9049	0.8790	0.8531	0.8272	0.8014	0.7755	0.7496
138,972	1.0296	1.0040	0.9783	0.9526	0.9270	0.9013	0.8756	0.8500	0.8243	0.7986	0.7730	0.7473
121,601	1.0268	1.0013	0.9758	0.9502	0.9247	0.8992	0.8737	0.8482	0.8227	0.7972	0.7717	0.7462
108,090	1.0250	0.9997	0.9743	0.9489	0.9235	0.8981	0.8727	0.8474	0.8220	0.7966	0.7712	0.7458
97,281	1.0240	0.9987	0.9735	0.9482	0.9229	0.8976	0.8723	0.8470	0.8217	0.7965	0.7712	0.7459
88,437	1.0231	0.9979	0.9727	0.9475	0.9223	0.8971	0.8719	0.8467	0.8215	0.7963	0.7711	0.7458
81,067	1.0225	0.9974	0.9722	0.9471	0.9220	0.8968	0.8717	0.8465	0.8214	0.7962	0.7711	0.7460
74,831	1.0218	0.9967	0.9716	0.9465	0.9214	0.8964	0.8713	0.8462	0.8211	0.7960	0.7709	0.7458
69,486	1.0221	0.9971	0.9720	0.9470	0.9219	0.8969	0.8718	0.8468	0.8218	0.7967	0.7717	0.7466
64,854	1.0215	0.9965	0.9715	0.9465	0.9215	0.8965	0.8715	0.8465	0.8215	0.7964	0.7714	0.7464
60,800	1.0213	0.9963	0.9713	0.9464	0.9214	0.8964	0.8715	0.8465	0.8215	0.7965	0.7716	0.7466
57,224	1.0205	0.9956	0.9707	0.9457	0.9208	0.8958	0.8709	0.8459	0.8210	0.7960	0.7711	0.7461
54,045	1.0204	0.9955	0.9706	0.9456	0.9207	0.8958	0.8709	0.8460	0.8210	0.7961	0.7712	0.7463
51,200	1.0201	0.9952	0.9703	0.9454	0.9205	0.8956	0.8707	0.8458	0.8209	0.7960	0.7711	0.7462
48,640	1.0191	0.9942	0.9693	0.9445	0.9196	0.8947	0.8698	0.8449	0.8201	0.7952	0.7703	0.7454

CONCLUSION

In this paper, the heuristic that supports buyer's multi-item replenishment decision when presented with a supplier's target discount contract has been addressed. The buyer's decision is to determine whether to order each item individually in an economic cycle or to order a group of items jointly in a target discount cycle. The buy quantities of all items in an individual buy are determined by the EOQ while in a joint buy are determined by balancing the time supply. The costs of the two cycles are calculated and compared based on the equal amount of buy quantities and, as a result, the cycle that provides the lower cost will be chosen. This heuristic is capable of solving large size problems. Also, its simplicity and performance are valuable to the buyer's replenishment decision making.

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